Notes_2_3_Basic_Set_Theory

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Notes_2_3_ Basic_Set...

2.2 Sample spaces

TERMINOLOGY: Suppose that a **random experiment** is performed and that we observe an outcome from the experiment (e.g., rolling a die). The set of all possible outcomes for an experiment is called the **sample space** and is denoted by S.

Example 2.2. In each of the following random experiments, we write out a corresponding sample space.

(a) The Michigan state lottery calls for a three-digit integer to be selected:

$$S = \{000, 001, 002, ..., 998, 999\}.$$

(b) A USC student is tested for chlamydia (0 = negative, 1 = positive):

$$S = \{0, 1\}.$$

(c) An industrial experiment consists of observing the lifetime of a battery, measured in hours. Different sample spaces are:

$$S_1 = \{w : w \geq 0\} \quad S_2 = \{0, 1, 2, 3, ..., \} \quad S_3 = \{\text{defective, not defective}\}.$$

Sample spaces are not unique; in fact, how we describe the sample space has a direct influence on how we assign probabilities to outcomes in this space. □

2.3 Basic set theory

TERMINOLOGY: A **countable set** A is a set whose elements can be put into a one-to-one correspondence with $\mathcal{N} = \{1, 2, ..., \}$, the set of natural numbers. A set that is not countable is said to be **uncountable**.

TERMINOLOGY: Countable sets can be further divided up into two types.

- A **countably infinite set** has an infinite number of elements.
- A **countably finite set** has a finite number of elements.

Example 2.3. Say whether the following sets are countable (and, furthermore, finite or infinite) or uncountable.

(a) $A = \{0, 1, 2, ..., 10\}$ countable, finite.

(b) $B = \{1, 2, 3, ..., \}$ countably infinite

(c) $C = \{x : 0 < x < 2\}$. (0,2) interval uncountable.

pos. integers < 4

$A = \{1, 2, 3\}$

$B = \{1, 2, 3, \ldots, 9\}$

$\Downarrow$

$A \cup B = \{1, \ldots, 9\}$

$A \cap B = \{1, 2, 3\}$

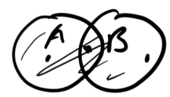
TERMINOLOGY: Suppose that A and B are sets (events). We say that A is a **subset** of B if every outcome in A is also in B, written $A \subset B$ or $A \subseteq B$.

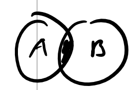
- IMPLICATION: In a random experiment, if the event A occurs, then so does B. The converse is not necessarily true.

TERMINOLOGY: The **null set**, denoted by $\emptyset$, is the set that contains no elements.

TERMINOLOGY: The **union** of two sets A and B is the set of all elements in either A or B (or both), written $A \cup B$. The **intersection** of two sets A and B is the set of all elements in both A and B, written $A \cap B$. Note that $A \cap B \subseteq A \cup B$.

- REMEMBER: Union $\longleftrightarrow$ "or" Intersection $\longleftrightarrow$ "and"





EXTENSION: We extend the notion of unions and intersections to more than two sets. Suppose that $A_1, A_2, ..., A_n$ is a **finite** sequence of sets. The union of $A_1, A_2, ..., A_n$ is

or or

$$x \text{ in} \bigcup_{j=1}^{n} A_j = A_1 \cup A_2 \cup \cdots \cup A_n,$$

that is, the set of all elements contained in at least one A_j. The intersection of $A_1, A_2, ..., A_n$ is

and x and

$$x \text{ in} \bigcap_{j=1}^{n} A_j = A_1 \cap A_2 \cap \cdots \cap A_n,$$

the set of all elements contained in each of the sets A_j, $j = 1, 2, ..., n$.

EXTENSION: Suppose that $A_1, A_2, ...,$ is a **countable** sequence of sets. The union and intersection of this infinite collection of sets is denoted by

$$\bigcup_{j=1}^{\infty} A_j \quad \text{and} \quad \bigcap_{j=1}^{\infty} A_j,$$

respectively. The interpretation is the same as before.

Example 2.4. Define the sequence of sets $A_j = [1-1/j, 1+1/j)$, for $j = 1, 2, ...,$. Then,

$$\bigcup_{j=1}^{\infty} A_j = [0, 2) \quad \text{and} \quad \bigcap_{j=1}^{\infty} A_j = \{1\}. \square$$

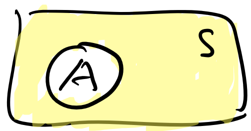
$A_1: [0, 2)$

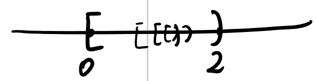
$A_2: [1-\frac{1}{2}, 1+\frac{1}{2}) = [\frac{1}{2}, \frac{3}{2})$

$A_3: [1-\frac{1}{3}, 1+\frac{1}{3}) = [\frac{2}{3}, \frac{4}{3})$

$\vdots$

TERMINOLOGY: Suppose that A is a subset of S (the sample space). The **complement** of a set A is the set of all elements not in A (but still in S). We denote the complement by $\overline{A}$.





Distributive Laws:

1. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
2. $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

DeMorgans Laws:

1. $\overline{A \cap B} = \overline{A} \cup \overline{B}$
2. $\overline{A \cup B} = \overline{A} \cap \overline{B}$

we prove: $\bigcap_{j=1}^{\infty} A_j = \{1\}$. obviously, 1 is in $\bigcap_{j=1}^{\infty} A_j$

If $x \neq 1$, $x \in \bigcap_{j=1}^{\infty} A_j$

Then, for all j we have

x is in $A_j = [1-\frac{1}{j}, 1+\frac{1}{j})$

Take j large enough such that $\frac{1}{j} < |x-1|$

then $x > 1+\frac{1}{j}$ or $x < 1-\frac{1}{j}$

i.e. x is not in A_j

Contradiction

So $\bigcap_{j=1}^{\infty} A_j$ can only be 1.

TERMINOLOGY: We call two events A and B **mutually exclusive**, or **disjoint**, if $A \cap B = \emptyset$, that is, A and B have no common elements.

Example 2.5. Suppose that a fair die is rolled. A sample space for this random experiment is $S = \{1, 2, 3, 4, 5, 6\}$.

(a) If $A = \{1, 2, 3\}$, then $\overline{A} = \{4, 5, 6\}$.

(b) If $A = \{1, 2, 3\}$, $B = \{4, 5\}$, and $C = \{2, 3, 6\}$, then $A \cap B = \emptyset$ and $B \cap C = \emptyset$. Note that $A \cap C = \{2, 3\}$. Note also that $A \cap B \cap C = \emptyset$ and $A \cup B \cup C = S$. $\square$

$A \cap B \cap C = \emptyset \;\not\Rightarrow\; \begin{cases} A \cap B = \emptyset \\ B \cap C = \emptyset \\ A \cap C = \emptyset \end{cases}$

$\Leftarrow$

pairwise disjoint