Notes_2_5 _Discrete_Probability_Models_and_Events

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Notes_2_5 _Discrete...

In general, the inclusion-exclusion formula can be written for any finite sequence:

$$P\left(\bigcup_{i=1}^{n} A_j\right) = \sum_{i=1}^{n} P(A_i) - \sum_{i_1<i_2} P(A_{i_1} \cap A_{i_2}) + \sum_{i_1<i_2<i_3} P(A_{i_1} \cap A_{i_2} \cap A_{i_3}) - \cdots + (-1)^{n+1} P(A_1 \cap A_2 \cap \cdots \cap A_n).$$

Of course, if the sets $A_1, A_2, ..., A_n$ are **pairwise disjoint**, then we arrive back at

$$P\left(\bigcup_{i=1}^{n} A_i\right) = \sum_{i=1}^{n} P(A_i),$$

a result implied by Axiom 3 by taking $A_{n+1} = A_{n+2} = \cdots = \emptyset$.

2.5 Discrete probability models and events

TERMINOLOGY: If a sample space for an experiment contains a finite or countable number of sample points, we call it a **discrete sample space**.

- **Finite:** "number of sample points $< \infty$."
- **Countable:** "number of sample points may equal ∞, but can be counted; i.e., sample points may be put into a 1:1 correspondence with $\mathcal{N} = \{1, 2, ..., \}$."

Example 2.7. A standard roulette wheel contains an array of numbered compartments referred to as "pockets." The pockets are either red, black, or green. The numbers 1 through 36 are evenly split between red and black, while 0 and 00 are green pockets. On the next play, we are interested in the following events:

$$A_1 = \{13\}$$
$$A_2 = \{\text{red}\}$$
$$A_3 = \{0, 00\}.$$

Suppose Equal chance

$38 \quad \frac{1}{38}$

$P(A_1) = P(\{13\}) = \frac{1}{38}$

$P(A_2) = \frac{18}{38}$

$P(A_3) = \frac{1}{38} + \frac{1}{38}$

$= \frac{2}{38}$

TERMINOLOGY: A **simple event** is an event that can not be decomposed. That is, a simple event corresponds to exactly one sample point. **Compound events** are those events that contain more than one sample point. In Example 2.7, because A_1 contains only one sample point, it is a simple event. The events A_2 and A_3 contain more than one sample point; thus, they are compound events.

STRATEGY: Computing the probability of a compound event can be done by

(1) counting up all sample points associated with the event (this can be very easy or very difficult)

$P(A_3) = P(\{0, 00\}) = P(\{0\}) + P(\{0.0\})$

(2) adding up the probabilities associated with each sample point.

NOTATION: Your authors use the symbol E_i to denote the ith sample point (i.e., ith simple event). Thus, adopting the aforementioned strategy, if A denotes any compound event,

$$P(A) = \sum_{i:E_i \in A} P(E_i).$$

We simply sum up the simple event probabilities $P(E_i)$ for all i such that $E_i \in A$.

Example 2.8. *An equiprobability model.* Suppose that a discrete sample space S contains $N < \infty$ sample points, each of which are **equally likely**. If the event A consists of n_a sample points, then $P(A) = n_a/N$.

Proof. Write $S = E_1 \cup E_2 \cup \cdots \cup E_N$, where E_i corresponds to the ith sample point; $i = 1, 2, ..., N$. Then,

$$1 = P(S) = P(E_1 \cup E_2 \cup \cdots \cup E_N) = \sum_{i=1}^{N} P(E_i).$$

Now, as $P(E_1) = P(E_2) = \cdots = P(E_N)$, we have that

$$1 = \sum_{i=1}^{N} P(E_i) = NP(E_1),$$

and, thus, $P(E_1) = \frac{1}{N} = P(E_2) = \cdots = P(E_N)$. Without loss of generality, take $A = E_1 \cup E_2 \cup \cdots \cup E_{n_a}$. Then,

$$P(A) = P(E_1 \cup E_2 \cup \cdots \cup E_{n_a}) = \sum_{i=1}^{n_a} P(E_i) = \sum_{i=1}^{n_a} \frac{1}{N} = n_a/N. \quad \square$$

$$\begin{pmatrix} M1 & M2 \\ W1 & W2 \end{pmatrix}$$

Example 2.9. Two jurors are needed from a pool of 2 men and 2 women. The jurors are randomly selected from the 4 individuals. A sample space for this experiment is

$$S = \{(M1, M2), (M1, W1), (M1, W2), (M2, W1), (M2, W2), (W1, W2)\}.$$

What is the probability that the two jurors chosen consist of 1 male and 1 female?

SOLUTION. There are $N = 6$ sample points, denoted in order by $E_1, E_2, ..., E_6$. Let the event

$$A = \{\text{one male, one female}\} = \{(M1, W1), (M1, W2), (M2, W1), (M2, W2)\},$$

so that $n_A = 4$. If the sample points are equally likely (probably true if the jurors are randomly selected), then $P(A) = 4/6$. □

2.6 Tools for counting sample points

2.6.1 The multiplication rule

MULTIPLICATION RULE: Consider an experiment consisting of $k \geq 2$ "stages," where

$$\begin{aligned} n_1 &= \text{number of ways stage 1 can occur} \\ n_2 &= \text{number of ways stage 2 can occur} \\ &\vdots \\ n_k &= \text{number of ways stage } k \text{ can occur.} \end{aligned}$$

Then, there are

$$\prod_{i=1}^{k} n_i = n_1 \times n_2 \times \cdots \times n_k$$

different outcomes in the experiment.

Example 2.10. An experiment consists of rolling two dice. Envision stage 1 as rolling the first and stage 2 as rolling the second. Here, $n_1 = 6$ and $n_2 = 6$. By the multiplication rule, there are $n_1 \times n_2 = 6 \times 6 = 36$ different outcomes. □