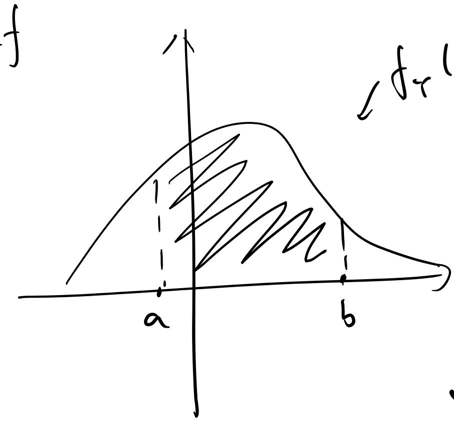
Univariate r.v. (Cont)

Y

pdf: $f_Y(y) \quad y \in \mathbb{R}$

cdf: $F_Y(y) = \underline{P(Y \le y)} = \int_{-\infty}^{y} f_Y(u)\,du$

pdf



$$\underline{P(a < Y < b) = \int_a^b f_Y(y)\,dy}$$

$P(Y_1 \le 1/3,\ Y_2 \le 1/2)$

$= \iint_B f_{Y_1,Y_2}(y_1, y_2)\, dy_2 dy_1$

$B = \{(y_1, y_2):\ 0 < y_1 \le 1/3,\ 0 < y_2 \le \frac{1}{2}\}$

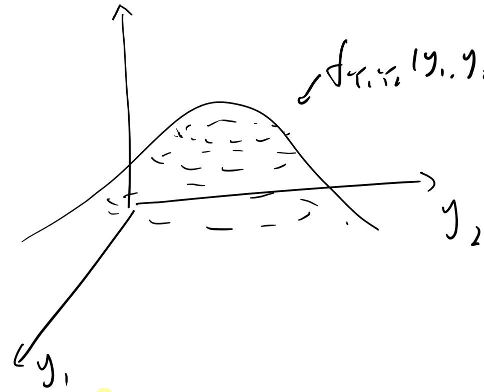
$= \int_0^{1/3} \int_0^{1/2} f_{Y_1,Y_2}(y_1, y_2)\, dy_2\, dy_1$

$P(Y_1 < Y_2)$

Continuous random vector

(Y_1, Y_2)

pdf: $f_{Y_1,Y_2}(y_1, y_2)$



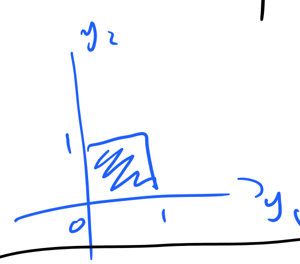
cdf: $F_{Y_1,Y_2}(y_1, y_2)$

$= \underline{P(Y_1 \le y_1,\ Y_2 \le y_2)}$

$= \int_{-\infty}^{y_1} \int_{-\infty}^{y_2} f_{Y_1,Y_2}(u_1, u_2)\, du_1\, du_2$

$$\frac{\partial^2 F_{Y_1,Y_2}(y_1, y_2)}{\partial y_1\, \partial y_2} = f_{Y_1,Y_2}(y_1, y_2)$$

Ex: $f_{Y_1,Y_2}(y_1, y_2) = \begin{cases} \frac{6}{5}(y_1 + y_2^2) & 0 < y_1 < 1,\ 0 < y_2 < 1 \\ 0 & \text{otherwise} \end{cases}$



$R = \{(y_1, y_2):\ 0 < y_1 < 1,\ 0 < y_2 < 1\}$

support of (Y_1, Y_2)

$P(Y_1 < Y_2)$

$B = \{ (y_1, y_2) : 0 < y_1 < y_2 < 1 \}$

$= \int_0^1 \int_{y_1}^1 f_{Y_1,Y_2}(y_1, y_2)\, dy_2\, dy_1$

$= \int_0^1 \int_0^{y_2} f_{Y_1 Y_2}(y_1, y_2)\, dy_1\, dy_2$

$P(Y_1 + Y_2 < 0.5)$

$B = \{ (y_1, y_2) : 0 < y_1 < 0.5 - y_2, \; 0 < y_2 < 0.5 \}$

$P(Y_1 Y_2 < 0.5)$

$B = \{ (y_1, y_2) : 0 < y_1 < \frac{0.5}{y_2} < 1, \; 0.5 < y_2 < 1 \}$

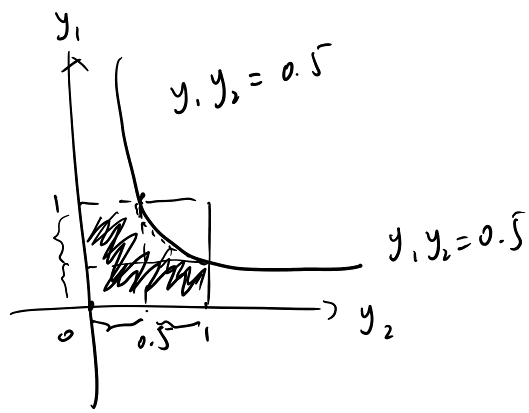
$\cup \{ (y_1, y_2) : 0 < y_1 < 1, \; 0 < y_2 < 0.5 \}$

$P(Y_1 + Y_2 < 1)$

$B = \{ (y_1, y_2) : y_1 + y_2 < 1, \; 0 < y_1 < 1, \; 0 < y_2 < 1 \}$

$= \{ (y_1, y_2) : 0 < y_1 < 1 - y_2, \; 0 < y_2 < 1 \}$

$\int_0^1 \int_0^{1-y_2} f_{Y_1 Y_2}(y_1, y_2)\, dy_1\, dy_2$



Marginal distribution

Conditional distribution

(Y_1, Y_2)

$\dots = \int^{y_1} \int^{+\infty} f_{Y_1 Y_2}(u_1, u_2)$

$$F_{Y_1}(y_1) = P(Y_1 \le y_1) = P(Y_1 \le y_1, -\infty < Y_2 < +\infty) = \int_{-\infty}^{y_1} \int_{-\infty}^{+\infty} f_{Y_1Y_2}(u_1,u_2)\, du_2\, du_1$$

$$= \int_{-\infty}^{y_1} \left[\int_{-\infty}^{+\infty} f_{Y_1Y_2}(u_1,u_2)\, du_2 \right] du_1$$

$$= \int_{-\infty}^{y_1} f_{Y_1}(u_1)\, du_1$$

$F_{Y_2}(y_2)$

$$f_{Y_1}(u_1) = \int_{-\infty}^{+\infty} f_{Y_1Y_2}(u_1,u_2)\, du_2$$

$$f_{Y_2}(u_2) = \int_{-\infty}^{+\infty} f_{Y_1,Y_2}(u_1,u_2)\, du_1$$