1. Bioterrorism experts are always trying to assess the impact of a future terrorist attack. Suppose that, in a hypothetical bioterrorist attack on New York City, 20 percent of people would possess symptom A; 30 percent would possess symptom B. Furthermore, 10 percent would possess both symptoms. What is the probability that a randomly selected person
(a) possesses at least one symptom?
(b) possesses symptom A, given that he has symptom B?
(c) possesses symptom A, but not B?
(d) possesses symptom B, given that he has at least one symptom?

2. Biologists are often interested in studying bacteria colonies in water samples. Suppose that Y denotes the number of colonies in a 1-cubic-centimeter specimen. For a particular bacterium, past experience says that Y is well-modeled by a Poisson distribution with mean $\lambda = 2.7$.
(a) What is the probability that a 1-cubic-centimeter specimen will contain **no bacteria colonies**?
(b) If four 1-cubic-centimeter specimens are to be analyzed, find the probability that **at least one** specimen will contain no bacteria colonies. It is logical to assume the four specimens are independent.
(c) If W denotes the number of specimens needed to observe the **first** specimen with no bacteria colonies, write out the probability mass function (pmf) of W. Be specific. It is (again) logical to assume that the specimens are independent.
(d) Use your pmf in part (c) to compute $P(W > 10)$.
Note: Treat (b) and (c) as different questions; they are not related.

3. The performance of compressor blades in jet engines is a critical issue to engineers. Historical evidence suggests that for a particular blade, Y, its operational lifetime (in 100s of hours), follows a Weibull distribution with probability density function (pdf)

$$f_Y(y) = \begin{cases} y^2 e^{-y^3/3}, & 0 < y < \infty \\ 0, & \text{otherwise.} \end{cases}$$

(a) Find the cumulative distribution function (cdf) for Y and graph it.
(b) Compute the probability that a single blade will fail before 100 hours; that is, compute $P(Y < 1)$.
(c) Find $E(Y)$ and $V(Y)$. Simplify your answers as much as possible.
Hints: I think you'll find the $u = y^3$ substitution helpful; don't try to derive the mgf.

4. Let Y_1 and Y_2 have a joint probability density function (pdf) given by

$$f_{Y_1,Y_2}(y_1, y_2) = \begin{cases} 3y_1, & 0 < y_2 < y_1 < 1 \\ 0, & \text{otherwise.} \end{cases}$$

(a) Graph the support of Y_1 and Y_2 in the y_1-y_2 plane. Describe, in words, what the joint density $f_{Y_1,Y_2}(y_1, y_2)$ looks like.

(b) Find the conditional distribution of Y_1, given $Y_2 = 1/2$.
(c) Compute $P(Y_1 > 3/4)$ and $P(Y_1 > 3/4|Y_2 = 1/2)$.
(d) Are Y_1 and Y_2 independent? Explain.

5. An electronic system has two different types of components in joint operation. Let Y_1 and Y_2 denote the random length of life (in hundreds of hours) of the components of Type I and Type II, respectively. Suppose that the joint probability density function (pdf) is given by

$$f_{Y_1,Y_2}(y_1, y_2) = \begin{cases} (1/8)y_1 e^{-(y_1+y_2)/2}, & y_1 > 0,\ y_2 > 0 \\ 0, & \text{otherwise.} \end{cases}$$

(a) Show that Y_1 and Y_2 are independent.
(b) Find $E(Y_1 + Y_2)$ and $V(Y_1 + Y_2)$.
(c) Find $E(\frac{Y_2}{Y_1})$.
(d) Express $P(Y_1 + Y_2 < 1)$ as a double integral; there is no need to evaluate it.

6. Suppose that Y_1, Y_2, and Y_3 are random variables with

$$\begin{array}{ccc} E(Y_1) = 0 & E(Y_2) = 1 & E(Y_3) = -1 \\ V(Y_1) = 1 & V(Y_2) = 2 & V(Y_3) = 3 \\ \text{Cov}(Y_1, Y_2) = -1 & \text{Cov}(Y_1, Y_3) = 0 & \text{Cov}(Y_2, Y_3) = 1. \end{array}$$

(a) Find $\text{Cov}(Y_1 + Y_2, Y_1 - Y_3)$.
(b) Define $U = 2Y_1 - Y_2 + Y_3$. Find the mean and variance of U.

7. Suppose that Y_1 and Y_2 are continuous random variables with joint pdf $f_{Y_1,Y_2}(y_1, y_2)$. The **joint moment generating function** of Y_1 and Y_2 (when it exists) is defined to be

$$m_{Y_1,Y_2}(t_1, t_2) \equiv E\left(e^{t_1 Y_1 + t_2 Y_2}\right) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{t_1 y_1 + t_2 y_2} f_{Y_1,Y_2}(y_1, y_2) dy_2 dy_1,$$

where t_1 and t_2 are non-zero constants.
(a) Prove that

$$E(Y_1 Y_2) = \left[\frac{\partial^2 m_{Y_1,Y_2}(t_1, t_2)}{\partial t_1 \partial t_2}\right]\Bigg|_{t_1=t_2=0}.$$

Hint: Assume that you can move the mixed partial derivative "inside" the double integral above; i.e., assume that you can interchange differentiation and integration.
(b) Suppose that Y_1 and Y_2 are **independent**. Show that

$$m_{Y_1,Y_2}(t_1, t_2) = m_{Y_1}(t_1) m_{Y_2}(t_2),$$

where $m_{Y_1}(t_1)$ and $m_{Y_2}(t_2)$ are the marginal mgfs of Y_1 and Y_2, respectively.