1. During one week, Joe will receive 7 text messages.
(a) What is the probability that Joe will receive exactly one text message each day? State any assumptions you need to answer this question. I would start by characterizing the underlying sample space.
(b) If 30 percent of all of Joe's text messages are from Polly, find the probability that at least 2 of the 7 text messages during one week are from Polly. State any assumptions you need to answer this question.

2. A normalized measurement of color for automotive paint is always guaranteed to fall between -1 and 1. Specifically, the measurement Y is a random variable with pdf

$$f_Y(y) = \begin{cases} \frac{3}{4}(1-y^2), & -1 < y < 1 \\ 0, & \text{otherwise.} \end{cases}$$

(a) Find the cumulative distribution function of Y and graph it.
(b) Find the probability that Y is greater than or equal to 1/2.

3. Suppose that Y is a random variable with mean $\mu = E(Y)$, variance $\sigma^2 = V(Y)$, and moment-generating function $m_Y(t)$. Define $Z = a + bY$, where a and b are constants.
(a) Show that $E(Z) = a + b\mu$.
(b) Show that $V(Z) = b^2\sigma^2$.
(c) Show that $m_Z(t) = e^{at}m_Y(bt)$.

4. A purchaser of electrical components buys them in lots of size 10 from two different suppliers. It is her policy to inspect 3 components randomly from the lot and to accept the lot only if all 3 are nondefective.

- The purchaser buys 30 percent of her lots from Supplier 1; Supplier 1 lots always contain 4 defectives out of 10.
- The purchaser buys 70 percent of her lots from Supplier 2; Supplier 2 lots always contain 1 defective out of 10.

What is the probability that the next lot will be accepted?
Hint: Define A to be the event that the lot is accepted, B_1 to be the event that the next lot is from Supplier 1, and B_2 to be the event that the next lot is from Supplier 2. There are only 2 suppliers, so $\{B_1, B_2\}$ partitions the space of possible suppliers. You want to compute $P(A)$.

5. The lifetime Y (in 1000s of hours) of a certain type of 100-watt industrial strength light bulb is a random variable with pdf

$$f_Y(y) = \begin{cases} \frac{1}{2}e^{-y/2}, & 0 < y < \infty \\ 0, & \text{otherwise.} \end{cases}$$

(a) Find the probability that one randomly selected lightbulb will last longer than 3000 hours; that is, compute $P(Y > 3)$.
(b) Suppose that we continue observing lightbulbs until we find the **first** one whose lifetime exceeds 3000 hours. What is the probability that we will need to observe **no more than** 4 lightbulbs?
(c) Suppose that we continue observing lightbulbs until we find the **third** one whose lifetime exceeds 3000 hours. What is the probability that we will have to observe **at least** 5 lightbulbs?
Note: In parts (b) and (c), assume that all lightbulbs are independent with the same probability from part (a).

6. Suppose that Y is a random variable with mean μ and variance σ^2. Recall that the skewness and kurtosis for a random variable Y are defined as

$$\xi = E[(Y-\mu)^3]/\sigma^3 \quad \text{(skewness)},$$

and

$$\kappa = E[(Y-\mu)^4]/\sigma^4 \quad \text{(kurtosis)},$$

respectively.
(a) If Y is a **standard normal** random variable; i.e., $Y \sim \mathcal{N}(0,1)$, show mathematically that $\xi = 0$ and $\kappa = 3$.
(b) For any random variable, describe to me, in words, what μ, σ^2, ξ, and κ measure. You can be brief.

7. The management at a fast-food outlet is interested in the joint behavior of the random variables Y_1 and Y_2. The variable Y_1 denotes the total time (in minutes) between a customers arrival at the store and his/her departure from the service window. The variable Y_2 denotes the time (in minutes) a customer waits in line before reaching the service window. Both Y_1 and Y_2 are measured in minutes. The joint distribution of Y_1 and Y_2 is given by

$$f_{Y_1,Y_2}(y_1,y_2) = \begin{cases} e^{-y_1}, & 0 < y_2 < y_1 < \infty \\ 0, & \text{otherwise.} \end{cases}$$

(a) Compute the probability that $Y_1 - Y_2$, the time spent at the service window, is greater than 1 minute; that is, compute $P(Y_1 - Y_2 > 1)$.
(b) Find both marginal distributions.
(c) Find the conditional distribution of Y_1 for customers whose value of $Y_2 = y_2 = 2.5$.

8. Let Y_1 and Y_2 denote the proportions of two different chemicals in a mixture of chemicals used as an insecticide. Suppose that (Y_1, Y_2) has the joint probability density function

$$f_{Y_1,Y_2}(y_1,y_2) = \begin{cases} 6y_1, & 0 \le y_1 \le 1,\ 0 \le y_2 \le 1,\ 0 \le y_1 + y_2 \le 1 \\ 0, & \text{otherwise.} \end{cases}$$

(a) The set $R = \{(y_1, y_2) : 0 \leq y_1 \leq 1,\ 0 \leq y_2 \leq 1,\ 0 \leq y_1 + y_2 \leq 1\}$ is the two-dimensional support set of (Y_1, Y_2). Sketch a picture of R.
(b) Describe what $f_{Y_1,Y_2}(y_1, y_2)$ looks like geometrically.
(c) Compute $\text{Cov}(Y_1, Y_2)$.
(d) Are Y_1 and Y_2 independent?