HW 5 (Due Nov 02, 2017) **Name:**

Problem 1. Finish HW 4 Problem 2 part (3).

Problem 2. Cosider the multiple linear regression model

$$\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\epsilon},$$

where $\mathbf{X}$ is $n \times p$ and $p = k + 1$. Let $\mathbf{M} = \mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'$ denote the hat matrix. Let $\mathbf{I}$ denoted the identity matrix that has the same dimensions as $\mathbf{M}$.

(a) What are the dimensions of $\mathbf{M}$?

(b) Show that both $\mathbf{M}$ and $\mathbf{I} - \mathbf{M}$ are symmetric and idempotent (a matrix $\mathbf{A}$ is idempotent if $\mathbf{A}^2 = \mathbf{A}$).

(c) Show that $\mathbf{MX} = \mathbf{X}$ and $(\mathbf{I} - \mathbf{M})\mathbf{X} = \mathbf{0}$.

(d) Show that $\mathbf{MY} = \mathbf{X}\widehat{\boldsymbol{\beta}}$

(e) Show that $(\mathbf{I} - \mathbf{M})\mathbf{Y} = \mathbf{e}$ where $\mathbf{e} = \mathbf{Y} - \widehat{\mathbf{Y}}$ and $\widehat{\mathbf{Y}} = \mathbf{X}\widehat{\boldsymbol{\beta}}$.

(f) Show that $(\mathbf{MY})'(\mathbf{I} - \mathbf{M})\mathbf{Y} = 0$.

(g) Show that $(\mathbf{Y} - \mathbf{X}\widehat{\boldsymbol{\beta}})'(\mathbf{Y} - \mathbf{X}\widehat{\boldsymbol{\beta}}) = \mathbf{Y}'(\mathbf{I} - \mathbf{M})\mathbf{Y}$.

Problem 3.

Consider the following data set on Y and two independent variables x_1 and x_2:

Y	x_1	x_2
5	1	1
5	1	-1
6	-1	1
8	-1	-1

I want you to do the following parts **by hand**, and show all of your work. You can use R to check your work.

(a) Write the multiple linear regression model in matrix form; i.e., what are $\mathbf{Y}$, $\mathbf{X}$, $\boldsymbol{\beta}$ and $\boldsymbol{\epsilon}$?

(b) Compute the least squares estimator $\widehat{\boldsymbol{\beta}}$.

(c) Find the covariance matrix of $\widehat{\boldsymbol{\beta}}$. What is the estimated standard error of $\widehat{\beta}_1$.

(d) Test $H_0 : \beta_1 = 0$ versuse $H_1 : \beta_1 \neq 0$ using $\alpha = 0.05$. What assumptions on the error $\boldsymbol{\epsilon}$ do you need for this hypothesis test to be valid?