STAT 712 Fall 2018 Exam I **Name:**____________________________

GROUND RULES:

1. The maximum points of this exam is 100.
2. Print your name at the top of this page in the upper right hand corner.
3. This is a closed-book and closed-notes exam. Show all of your work and explain all of your reasoning.
4. Any discussion or other inappropriate communication between examinees, as well as the appearance of any unnecessary materials, will be dealt with severely.
5. Keep 4 decimal places if needed.
6. You have 80 minuets to complete this exam. GOOD LUCK!

HONOR PLEDGE FOR THIS EXAM: After you have finished the exam, please read the following statement and sign your name below it.

I promise that I did not discuss any aspect of this exam with anyone other than the instructor, that I neither gave nor received any unauthorized assistance on this exam, and that the work presented herein is entirely my own. Signature:____________________________

1. (Under the similar setting as the troop-loading problem in HW). There are **five** different types of troop (tank, rifleman, zooka, heavy, and warrior) a player will **randomly** choose to fill up **five** identical landing crafts. Each craft can take one and only one type of troop. By "fill up" it means the maximum capacity of each craft for the chosen troop is always automatically reached. Let X denote the number of crafts which are filled up by warriors. Find the probability mass function of X and sketch the corresponding cumulative distribution function (*please clearly explain your reasoning and show all the work. When sketching the cdf, please highlight the right continuous feature of a cdf in your plot*).

2. Let X be a continuous random variable with $E|X| < \infty$ and cumulative distribution function F_X.

(a) Show that

$$E(X) = \int_0^\infty \{1 - F_X(t)\}dt - \int_{-\infty}^0 F_X(t)dt.$$

(b) Let Y be another continuous random variable with cumulative distribution function F_Y. A HW problem (1.49) introduced that X is *stochastically greater than* Y if $F_X(t) \leq F_Y(t)$ for all t. Now suppose $F_X(t) \leq F_Y(t)$ indeed holds for all t. Prove that

$$E(X) \geq E(Y).$$

3. Let P_n denote the probability that successive "6"s never appear when a dice is tossed n times. Find

$$\lim_{n\to\infty} \frac{P_n}{P_{n-1}}.$$

4. Let the random variable X have the pdf

$$f_X(x) = \frac{1}{2\beta} \exp\left(-\frac{|x-\alpha|}{\beta}\right), \quad \text{for } -\infty < x < \infty.$$

(a) Find the moment generating function, $M_X(t)$, of X (make sure to note for which values of t the function is defined).

(b) Use $M_X(t)$ to find $E(X)$ and $\mathrm{var}(X)$.

(c) Find the density function of $Y = \sqrt{2|X-\alpha|/\beta}$.

5. For a given sample space $\mathcal{S}$, let $\mathcal{C}$ be a collection of subsets of $\mathcal{S}$. Denote by $\sigma(\mathcal{C})$ the intersection of all σ-algebras on $\mathcal{S}$ containing $\mathcal{C}$. Prove $\sigma(\mathcal{C})$ is also a σ-algebra, and is the smallest σ-algebra on $\mathcal{S}$ containing $\mathcal{C}$ (by "smallest", it means $\sigma(\mathcal{C})$ is a subset of any other σ-algebra on $\mathcal{S}$ containing $\mathcal{C}$).