STAT 712 Fall 2018 Exam II **Name:**____________________________

GROUND RULES:

1. The maximum points of this exam is 100.
2. Print your name at the top of this page in the upper right hand corner.
3. This is a closed-book and closed-notes exam. Show all of your work and explain all of your reasoning.
4. Any discussion or other inappropriate communication between examinees, as well as the appearance of any unnecessary materials, will be dealt with severely.
5. Keep 4 decimal places if needed.
6. You have 80 minuets to complete this exam. GOOD LUCK!

HONOR PLEDGE FOR THIS EXAM: After you have finished the exam, please read the following statement and sign your name below it.

I promise that I did not discuss any aspect of this exam with anyone other than the instructor, that I neither gave nor received any unauthorized assistance on this exam, and that the work presented herein is entirely my own. Signature:____________________________

1. Suppose that X as a beta(α, β) distribution. The pdf of X is

$$f_X(x|\alpha,\beta) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1}(1-x)^{\beta-1} I(0 < x < 1).$$

(a) Show that $\{f_X(x|\alpha,\beta); \alpha > 0, \beta > 0\}$ is a two-parameter exponential family. Is it a full or curved family? Why?

(b) Define

$$Y = \frac{X}{1-X}.$$

Derive the pdf of Y. Also find $E(Y)$ for $\beta > 1$.

(c) Prove that the mgf of Y does not exist.

2. In general, if we say $W \sim N(a, b^2)$, then the pdf of W is

$$f_W(w) = \frac{1}{\sqrt{2\pi b^2}} \exp\left\{-\frac{(w-a)^2}{2b^2}\right\} I(-\infty < w < \infty).$$

Now let $X \sim N(\mu, \sigma^2)$ and $Y \sim N(\gamma, \sigma^2)$. Suppose the X and Y are independent. Define $U = X+Y$, $V = X - Y$.

(a) Are U and V independent? If yes, prove it; if no, explain why.

(b) Find the distribution of U and also the distribution of V.

3. Consider the following hierarchical model:

$$X \mid Y \sim \text{exponential}(1/Y)$$
$$Y \sim \text{gamma}(a, b).$$

The pdf of an exponential or gamma distribution can be found on the formula sheet.

(a) Find the pdf of X.

(b) Calculate $E(X)$ and $\text{var}(X)$. Note any restrictions on the values of a and b.

(c) Derive $f_{Y|X}(y|x)$ and find $E(Y|X = x)$.