Chapter 7. Point Estimation.

$$X_1, \cdots, X_n \overset{iid}{\sim} f_X(x \mid \underset{\sim}{\theta}) \qquad \underset{\sim}{\theta} \subseteq R^k$$

$$\underset{\sim}{X} = (X_1, \cdots, X_n) \sim \underline{\prod_{i=1}^{n} f_X(x_i \mid \theta)} = f_{\underset{\sim}{X}}(\underset{\sim}{x} \mid \theta)$$

where $\underset{\sim}{x} = (x_1, \cdots, x_n)$

Goal:

- Estimate $\underset{\sim}{\theta}$
- Estimate $\tau(\underset{\sim}{\theta})$ where $\tau : \mathbb{R}^k \to \mathbb{R}^q$ $(q \le k)$

Definition: A Point Estimator of θ for $\tau(\theta)$

$\tau(\hat{\theta})$

, denoted by $\hat{\theta}$, which is $W(\underset{\sim}{X}) = W(X_1, \cdots, X_n)$

$\widehat{\tau(\theta)}$

a function of the sample $\underset{\sim}{X}$.

Estimator

Estimate: $W(x_1, \cdots, x_n)$ observed!

How to find $\hat{\theta}$?

- Method of Moments (MoM)
- Maximum Likelihood Estimation (MLE)
- Bayesian Estimation

• MoM.

WLLN: iid r.v. Y_i, $i=1,\dots,n \sim Y$ ($Var(Y)<\infty$)

$$\frac{1}{n}\sum Y_i \xrightarrow{P} E[Y]$$

$$X_1,\dots,X_n \overset{iid}{\sim} f_X(x|\underset{\sim}{\theta}) \qquad \underset{\sim}{\theta}=(\theta_1,\dots,\theta_k)\in R^k$$

$$E[X] = g_1(\theta_1,\dots,\theta_k)$$

$$E[X^2] = g_2(\theta_1,\dots,\theta_k)$$

$$\vdots$$

$$E[X^k] = g_k(\theta_1,\dots,\theta_k)$$

$$N(\mu,\sigma^2) \qquad \underset{\sim}{\theta} = \begin{pmatrix}\mu\\ \sigma^2\end{pmatrix}$$

$$E[X] = \mu = g_1(\underset{\sim}{\theta}) = g_1(\mu,\sigma^2) \xleftarrow{P} \frac{1}{n}\sum X_i$$

$$E[X^2] = \sigma^2+\mu^2 = g_2(\mu,\sigma^2) \xleftarrow{P} \frac{1}{n}\sum X_i^2$$

Set: $\begin{cases} \frac{1}{n}\sum X_i = g_1(\mu,\sigma^2) = \mu \\ \frac{1}{n}\sum X_i^2 = g_2(\mu,\sigma^2) = \sigma^2+\mu^2 \end{cases}$

Solve it for μ and σ^2

$$\left.\begin{aligned} \hat{\mu} &= \bar{X}_n \\ \hat{\sigma}^2 &= \frac{1}{n}\sum X_i^2 - \bar{X}_n^2 \end{aligned}\right\} \text{MoM Estimator of } \mu \text{ and } \sigma^2$$

not unique

$$E[X^2] = \sigma^2 + \mu^2 \quad \leftarrow \frac{1}{n}\sum X_i^2$$

$$E[X^3] = \ldots\ldots \quad \leftarrow \frac{1}{n}\sum X_i^3$$

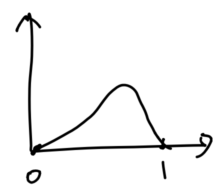
Maximum Likelihood Estimation !

10 flip 8 heads 2 tails

p : prob of a head

$L(p)$

$$P(\text{observing the data}) = \binom{10}{8} p^8 (1-p)^2$$



which $\hat{p}$ is a better guess of p

$\hat{p} = 0.1$ $\hat{p} = 0.5$ $\hat{p} = 0.8$ $\hat{p} = 0.9$

Likelihood function. $X_1, \ldots, X_n \sim f_X(x|\underset{\sim}{\theta})$ $\underset{\sim}{\theta} \in \mathbb{R}^k$

$$L(\underset{\sim}{\theta}) = \prod_{i=1}^{n} f_X(X_i | \underset{\sim}{\theta}) \qquad \underset{\sim}{\theta} \in \Theta$$

Maximum Likelihood Estimator.

is the $\hat{\theta}$ that maximizes $L(\underset{\sim}{\theta})$ in Θ