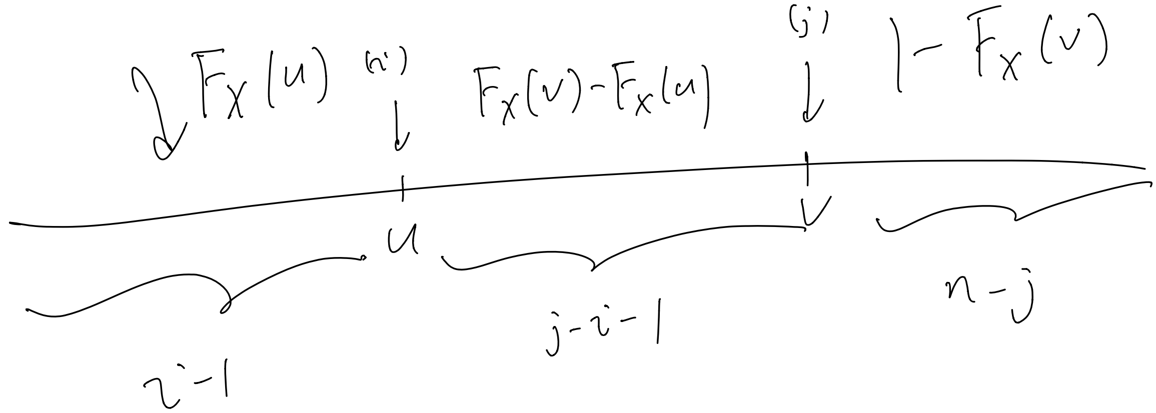
March 26, 2019

$$f_{X_{(j)}}(x) = \frac{n!}{(n-j)!\,(j-1)!} \left[F_X(x)\right]^{j-1} f_X(x) \left[1 - F_X(x)\right]^{n-j}$$

$i < j \qquad u < v$



$$i-1 + 1 + j-i-1 + 1 + n-j = n$$

$$f_{X_{(i)}, X_{(j)}}(u, v) = \frac{n!}{(i-1)!\,(j-i-1)!\,(n-j)!} \times \left[F_X(u)\right]^{i-1} \cdot f_X(u)$$

$$\times \left[F_X(v) - F_X(u)\right]^{j-i-1} \times f_X(v)$$

$$\times \left[1 - F_X(v)\right]^{n-j}, \quad u < v$$

$u_1 < u_2 < \cdots < u_n$

$$f_{X_{(1)} \cdots X_{(n)}}(u_1, \ldots, u_n)$$

$$= n!\, f_X(u_1) \cdots f_X(u_n)$$