**HW 2 (Due Mar. 18, 2015)**

1. Let $\{Y_t\}$ be a stationary zero-mean time series. Define

$$X_t = Y_t - 0.4Y_{t-1}$$

and

$$W_t = Y_t - 2.5Y_{t-1}$$

Express the ACVF of $\{X_t\}$ and $\{W_t\}$ in terms of the ACVF of $\{Y_t\}$. Further show that the autocorrelation functions of $X_t$ and $W_t$ are the same.

2. Find the coefficients $\psi_j, j = 0, 1, 2, \ldots$ in the representation

$$X_t = \sum_{j=0}^{\infty} \psi_j W_{t-j}$$

of the ARMA(2,1) process,

$$(1 - 0.5B + 0.04B^2)X_t = (1 + 0.25B)W_t, W_t \sim \text{WN}(0, \sigma^2).$$

3. For an MA(2) process, find the largest possible values of $|\rho(1)|$ and $|\rho(2)|$.

4. Find the autocovariances $\gamma_X(h)$, $h = 0, 1, 2, \ldots$ of the AR(3) process,

$$(1 - 0.5B)(1 - 0.4B)^2 X_t = W_t, W_t \sim \text{WN}(0, \sigma^2).$$

5. Suppose $\{X_t\}$ is the two sided moving average

$$X_t = \sum_{j=-\infty}^{\infty} \psi_j W_{t-j}, W_t \sim \text{WN}(0, \sigma^2)$$

where $\sum_j |\psi_j| < \infty$. Show that

$$\sum_{h=-\infty}^{\infty} |\gamma_X(h)| < \infty.$$

6. Find the spectral density of an MA(1) process, $X_t = W_t + \theta W_{t-1}, W_t \sim \text{WN}(0, \sigma^2)$ and of an AR(1) process, $X_t - \phi X_{t-1} = W_t, W_t \sim \text{WN}(0, \sigma^2)$

7. If $\{X_t\}$ and $\{Y_t\}$ are stationary processes satisfying

$$X_t - \alpha X_{t-1} = W_t, W_t \sim \text{WN}(0, \sigma^2)$$

and

$$Y_t - \alpha Y_{t-1} = X_t + Z_t, Z_t \sim \text{WN}(0, \sigma^2)$$

where $|\alpha| < 1$ and $\{W_t\}$ and $\{Z_t\}$ are uncorrelated, find the spectral density of $\{Y_t\}$.

8. For the MA(1) process $X_t = W_t + \theta W_{t-1}$, $|\theta| < 1$, $W_t \sim \text{WN}(0, \sigma^2)$. Find its PACF and

$$\nu_n = E\{X_{n+1} - \overline{\mathbb{P}}(X_{n+1} \mid X_1, \ldots, X_n)\}^2.$$

Further find the limit of $\nu_n$ when $n$ goes to infinity.

9. If $\{X_1, \ldots, X_n\}$ are observations of the AR($p$) process,

$$X_t - \phi_1 X_{t-1} - \cdots - \phi_p X_{t-p} = W_t, W_t \sim \text{WN}(0, \sigma^2),$$

show that

$$E\{\overline{\mathbb{P}}(X_{n+h} \mid X_1, \ldots, X_n) - X_{n+h}\}^2 = \sigma^2 \sum_{j=0}^{h-1} \psi_j^2, \quad \text{for } n \geq p, h \geq 1,$$

where $\psi(z) = \sum_{j=0}^{\infty} \psi_j z^j = 1/\phi(z)$.