HW 3 (Due April. 15, 2015)

1. Show that if $n > p$, the likelihood of the observations $\{X_1, \ldots, X_n\}$ of the causal AR(p) process,

$$X_t - \phi_1 X_{t-1} - \cdots - \phi_p X_{t-p} = W_t, \quad W_t \sim N(0, \sigma^2),$$

   is

$$L(\boldsymbol{\phi}, \sigma^2) = (2\pi\sigma^2)^{-n/2}(\det G_p)^{-1/2} \times \exp\left\{-\frac{1}{2}\sigma^{-2}\left[\boldsymbol{X}_p' G_p^{-1} \boldsymbol{X}_p + \sum_{t=p+1}^{n} (X_t - \phi_1 X_{t-1} - \cdots - \phi_p X_{t-p})^2\right]\right\}$$

   where $\boldsymbol{X}_p = (X_1, \ldots, X_p)'$ and $G_p = \sigma^{-2} E(\boldsymbol{X}_p \boldsymbol{X}_p')$. Further given two observations x_1 and x_2 from the causal AR(1) process

$$X_t - \phi X_{t-1} = W_t, \quad W_t \sim N(0, \sigma^2)$$

   such that $|x_1| \neq |x_2|$, find the MLE of ϕ and σ^2.

2. For an AR(p) process, show that $\det \Gamma_m = (\det \Gamma_p)\sigma^{2(m-p)}$ for all $m > p$. Conclude that the (m, m) component of Γ_m^{-1} is $(\det \Gamma_{m-1})/(\det \Gamma_m) = \sigma^{-2}$. (This proves the $\sqrt{n}\widehat{\phi}_{mm} \to^d N(0, 1)$ in Theorem 6.2)

3. Suppose that $\{X_t\}$ is an ARIMA(p, d, q) process, satisfying the difference equations

$$\phi(B)(1 - B)^d X_t = \theta(B) W_t, \quad W_t \sim \mathrm{WN}(0, \sigma^2)$$

   show that these difference equations are also satisfied by the process

$$Z_t = X_t + \alpha_0 + \alpha_1 t + \cdots + \alpha_{d-1} t^{d-1}$$

   for arbitrary random variables $\alpha_0, \alpha_1, \ldots, \alpha_{d-1}$.

4. Let $\{X_t\}$ be the seasonal process,

$$(1 - .7B^2) X_t = (1 + .3B^2) W_t, \quad W_t \sim \mathrm{WN}(0, \sigma^2)$$

   Find the coefficients $\{\psi_j\}$ such that $X_t = \sum_{j=0}^{\infty} \psi_j W_{t-j}$. Further, find the coefficients $\{\pi_j\}$ such that $W_t = \sum_{j=0}^{\infty} \pi_j X_{t-j}$.