**STAT 823, Homework 3, due Feb. 25, 2020**

1. *Runs of increasing values.* Let $X_0, X_1, X_2, \ldots$ be i.i.d. random variables from a continuous distribution, $F(x)$. Let $Z_j$ be one if there is a relative minimum at $j$, and zero otherwise; that is $Z_j = I(X_{j-1} > X_j < X_{j+1})$. Then $T_n = \sum_{j=1}^{n} Z_j$ represents the number of relative minima in the sequence of $X_i$'s. Find the asymptotic distribution of $T_n$.

2. Let $X_1, X_2, \ldots$ be independent and suppose that $X_n = \sqrt{n}$ with probability 0.5 and $X_n = -\sqrt{n}$ with probability 0.5 for $n = 1, 2, \ldots$ Find the asymptotic distribution of $\bar{X}_n$. (Check the Lindeberg Condition).

3. *Records.* Let $Z_1, Z_2, \ldots$ be i.i.d. continuous random variables. We say a record occurs at $k$ if $Z_k > \max_{i<k} Z_i$. Let $R_k = 1$ if a record occurs at $k$, and let $R_k = 0$ otherwise. Show that $R_1, R_2, \ldots$ are independent Bernoulli random variables with $P(R_k = 1) = 1 - P(R_k = 0) = 1/k$ for $k = 1, 2, \ldots$. Let $T_n = \sum_{k=1}^{n} R_k$ denote the number of records in the first $n$ observations. Find $E(T_n)$ and $Var(T_n)$, and show that $(T_n - E\{T_n\})/\sqrt{Var(T_n)}$ converges to $N(0,1)$ in distribution.

4. Let $Y_1, Y_2, \ldots Y_{n+1}$ be i.i.d. exponential random variables with mean 1, and let $S_j = \sum_{i=1}^{j} Y_i$ for $j = 1, \ldots, n+1$. Then the conditional joint distribution of

$$\left( \frac{S_1}{S_{n+1}}, \ldots, \frac{S_n}{S_{n+1}} \right)$$

given $S_{n+1}$ is the same as the order statistics of a i.i.d. sample of size $n$ from Uniform$(0,1)$.