1. Let $X_1, X_2, \ldots$, be a sample from Uniform$(0, 2\mu)$.
   (a) Find the asymptotic distribution of the median.
   (b) Find the asymptotic distribution of the midquartile range; i.e., $\{X_{[3n/4]} + X_{[n/4]}\}/2$.
   (c) Find the asymptotic distribution of $2X_{[3n/4]}/3$.
   (d) Compare these three estimates of $\mu$.

2. Let $X_1, X_2, \ldots$ be a sample from the beta distribution with density $f(x; \theta) = \theta x^{\theta - 1} I(0 < x < 1)$, where $\theta > 0$,
   (a) Let $M_n$ denote the sample median and $m(\theta)$ the population median as a function of $\theta$. What is the asymptotic distribution of $\sqrt{n} \{M_n - m(\theta)\}$.
   (b) Let $\hat{\theta}_n = \log(1/2)/\log(M_n)$. Show $\hat{\theta}_n$ converges to $\theta$ in probability.
   (c) What is the asymptotic distribution of $\sqrt{n}(\hat{\theta}_n - \theta)$.

3. For the following distributions, find the normalization such that $(M_n - a_n)/b_n$ has a nondegenerate limit if any exists ($M_n$ is the large order statistics of a sample of size $n$).
   (a) $f(x) = e^x I(x < 0)$.
   (b) $f(x) = (2/x^3)I(x > 1)$.
   (c) $F(x) = 1 - \exp\{-x/(1 - x)\}$ for $0 < x < 1$.

4. Let $X_1, \ldots, X_n$ be a sample from Uniform$(\theta - 0.5, \theta + 0.5)$. Among the various estimates of $\theta$, one may use the sample median $\hat{\theta}_1$, and one may use the midrange, $\hat{\theta}_2 = (\max X_i + \min X_i)/2$. Compare the 95% confidence interval for $\theta$ obtained from these two estimates, when $n = 100$. 